

Modelling Physical Behaviour of the Unidirectional Composite Materials with FEM Using Reduced Data

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Abstract. The aim of the work is to develop multi-scale finite element (FE) models of unidirectional fibre composite materials. This enables to create the models of reasonable dimensionality, which are able to present adequately the mechanical behaviour of the highly complex internal micro-structure of the material. As a sample structure, the sheet of epoxy reinforced with fibres is considered. The homogenized material properties of the shell FE at the macro-scale are obtained by performing a proper sequence of numerical experiments on the refined finite element structure of the representative volume of the composite. The representative volume is a micro-cube, the FE model of which presents physical and geometrical properties at the micro-scale periodically recurring within the overall volume of the composite. The behaviour of the micro-cube is simulated including large deformation and failure. Linear material properties are obtained by using pure strains assumptions in the implicit analysis of the micro-cube, while the non-linear behaviour and failure parameters require the explicit dynamic analysis. Simulation is performed by using LS-DYNA finite element software.

Keywords: unidirectional composite, failure, effective properties.

1. Introduction

Composites are materials with complex internal structure composed of two or more materials of significantly different mechanical properties. Typical unidirectional (UD) composites consist of the epoxy matrix reinforced with fibres. The stiffness of the fibre material is much higher than the stiffness of the matrix (Vasiliev and Morozov, 2001). As composite materials are artificial, the desirable overall mechanical properties can be achieved by varying the properties and geometry of composing materials (Milton, 2002). Due to favourable strength against mass ratio the unidirectional composites are widely used to manufacture products for sports, aircraft, medical use, etc.

Simulation of the mechanical behaviour of the unidirectional fibre composite by taking into account its microstructure requires unrealistically large computational resources. The multi-scale approach to the modelling enables to present the model of the composite by reduced models of much lesser dimensionality, which simultaneously retain all main features of mechanical behaviour of the micro-structure of the composite. Numerous approaches have been developed in order to establish the physically adequate relationships between the models of an object presented at micro- and macro- scales.

Homogenization methods (Xing et al., 2010; Pinho-da-Cruz, 2009), analytical approach (Kanit et al., 2003; Vasiliev and Morozov, 2001; May et al., 2014), numerical simulations performed on the samples of the microstructure (Berger et al., 2005) were employed in order to evaluate the effective elastic properties of the composite material. For the random media, the sample of the microstructure must include all microstructural heterogeneities that occur in the composite (Kanit et al., 2003). In case of the ideal periodic structure a unit cell is able to capture the major features of the underlying microstructure (Berger et al., 2005) and the increase of the number of fibres in the model would not affect the results of the mechanical behaviour (Thibaux et al., 2000). Other conditions applied to define the size of the representative structure (RS) are discussed in Pelissou et al. (2009).

The stiffness and strength of the UD composite along the fibres are governed mostly by the fibres and are not perceptibly influenced by the cracks in the matrix. Under longitudinal tension the structure fails if the fibres fail. It follows that the strength of the fibres under tensile load can be employed to identify failure in the longitudinal direction using the maximum stress criterion (Ribeiro et al., 2012). Vasiliev and Morozov (2001) define longitudinal tensile strength as the stress at the ultimate elongation of the fibre for the material with the stiffness evaluated using the rules of mixture.

The stiffness and strength of the UD composite in the transverse and shear directions are influenced by both matrix and fibre materials. Usually the damage in the composite under the transverse tensile or shear load occurs in the matrix and in the fibre-matrix interface (Maimi et al., 2007). The 1st order model (FOM) of the UD composite contains one layer of fibres of circular cross-section aligned in parallel. For the FOM, the transverse strength is equal to the ultimate stress of the matrix material and the shear strength exceeds the transverse strength (Vasiliev and Morozov, 2001). However, shear strength is not evaluated analytically.

In this work the FOM is proposed to evaluate the effective elastic parameters analytically and using FEM simulation. Two approaches were applied to evaluate shear parameters of the material. The shear parameters evaluated using traditional approach with pure shear simulation depends on the size of the analysed structure. The new approach to evaluate shear parameters using shear simulation without the straight side requirement is presented in this article. Furthermore, numerical examples are presented in this article to compare the stresses of the heterogeneous and homogenized material models.

2. The first order model of the UD composite

The representative structure (RS) of the 1st order model (FOM) of the UD composite (Fig.1, B) consisting of the shell elements is used to evaluate the effective parameters of the composite material. The 4-node shell element (6 degrees of freedom per node) is employed for the simulation of the UD composite at the macro-scale with the assumption that the material is homogeneous (Fig.1, C). The periodic element of the FOM (Fig.1, B) consisting 81 nodes (6 degrees of freedom per node) derives from the solid model (Fig.1, A) which is composed of 735 nodes (3 degrees of freedom per node). As a result, degrees of freedom of the periodic element are reduced from 2205 for the solid model to 486 for the FOM and 24 for the homogeneous shell model.

The micro-structure of the unidirectional composite is defined by the fibre fraction v_f and the direction of the fibres only. Hence the composing material can be presented as a system of the areas simulating fibres and matrix with the respective fraction in the structure (Vasiliev and Morozov, 2001):

$$v_f = \frac{a_f}{a}, v_m = \frac{a_m}{a}, a = a_m + a_f \quad (1)$$

a_f, a_m are the dimensions of the fibre and matrix on the side of the periodic element (Fig.1, B), v_f, v_m – fraction of the fibre and matrix in the model.

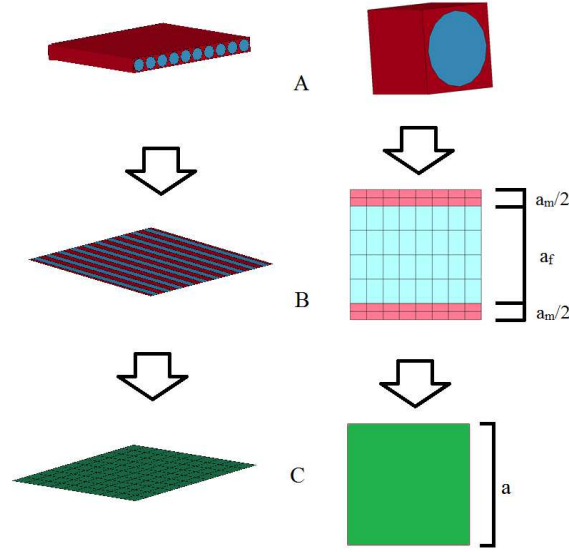


Fig.1. Scheme for data reduction employed to simulate the mechanical behaviour of the UD composite: solid model (A), the 1st order shell model (B), the homogeneous shell model (C). 10x10 structures of the models are shown on the left, the periodic elements are shown on the right.

2.1. Analytical evaluation of the effective parameters

The effective parameters of FOM are evaluated analytically using the following equations (Vasiliev and Morozov, 2001):

$$E_x = E_f v_f + E_m v_m \quad (2)$$

$$E_y = \frac{1}{\frac{v_f}{E_f} + \frac{v_m}{E_m} - \frac{v_f v_m (E_f v_m - E_m v_f)^2}{E_f E_m (E_f v_f + E_m v_m)}} \quad (3)$$

$$v_{yx} = \frac{(v_f v_f + v_m v_m) E_y}{E_x} \quad (4)$$

$$G_{xy} = \frac{1}{\frac{v_f}{G_f} + \frac{v_m}{G_m}} \quad (5)$$

where E_{Ω} – Young's modulus, ν_{Ω} – Poisson's ration, G_{Ω} –shear modulus. Subscripts f, m indicate the parameters of the fibre or matrix material, and subscripts x, y, xy refer to the direction of the effective parameter of the homogeneous material.

If the fibre material is linear until the failure and $\bar{\varepsilon}_f$ is the ultimate elongation of the fibre (usually less than the ultimate elongation of the matrix material $\bar{\varepsilon}_m$), the longitudinal tensile strength XT is determined as (Vasiliev and Morozov, 2001):

$$XT = (E_f v_f + E_m v_m) \bar{\varepsilon}_f \quad (6)$$

If the stiffness reduction after yielding is considered, the longitudinal tensile strength is calculated as follows:

$$XT = v_f (E_f \varepsilon_{Yf} + ET_f (\bar{\varepsilon}_f - \varepsilon_{Yf})) + v_m (E_m \varepsilon_{Ym} + ET_m (\bar{\varepsilon}_f - \varepsilon_{Ym})) \quad (7)$$

$$\varepsilon_{Yf} \leq \bar{\varepsilon}_f, \varepsilon_{Ym} \leq \bar{\varepsilon}_f, \bar{\varepsilon}_m \leq \bar{\varepsilon}_f$$

$ET_{f,m}$ – tangent modulus of the fibre or matrix material, $\varepsilon_{Yf,m}$ – the strain at the yield point of the respective material and product $E\varepsilon_Y$ is equal to the yield stress.

For the FOM the transverse tensile strength YT is determined as the ultimate stress of the matrix material $\bar{\sigma}_m$ (Vasiliev and Morozov, 2001):

$$YT = \bar{\sigma}_m \quad (8)$$

The shear stress-strain relation is non-linear and the ultimate shear strength SC exceeds the ultimate transverse tensile strength YT for the UD composite. In addition, the tensile strength in the transverse direction and shear strength depend on the strain rate and the longitudinal strength is independent of the strain rate (Taniguchi et al., 2012).

2.2. FEM approach for the identifying the effective parameters

FEM simulation of the representative structure (RS) is applied in order to obtain the homogenized elastic constants. The RS of FOM (Fig.1, B) is subjected to deformations by applying the load schemes presented in Fig.2: uniaxial (Fig.2 A, B) and pure shear (Fig.2 C). The sides of the RS remain straight in the deformed configuration.

The implicit FE analysis is employed to evaluate the linear elastic constants. The loading is performed by prescribing small displacements of the nodes on the edges of the RS in accordance with the schemes in Fig.2. Implicit simulation is performed by using LS-DYNA FE software. The mean stresses of the RS are calculated as follows:

$$\sigma = \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix} = \sum_i \frac{V_i}{V} \begin{Bmatrix} \sigma_{xx}^{\{i\}} \\ \sigma_{yy}^{\{i\}} \\ \tau_{xy}^{\{i\}} \end{Bmatrix} \quad (9)$$

V_i – area of the i-th element, V – area of the RS, $\sigma_{xx}^{\{i\}}$ – stress of the i-th element in the longitudinal direction, $\sigma_{yy}^{\{i\}}$ – stress of the i-th element in the transverse direction, $\tau_{xy}^{\{i\}}$ – shear stress of the i-th element.

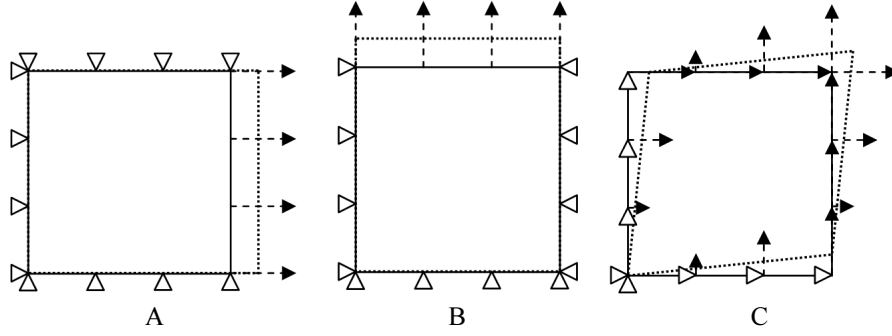


Fig.2. Load schemes for the RS: A – longitudinal tension mode; B – transverse tension mode; C – shear mode. Empty triangles define constraints (symbols $\triangleright \triangleleft$ mean that the node is constrained in x direction and symbols $\triangle \nabla$ mean that the node is constrained in y direction). Dotted lines present the deformed configuration.

The elastic compliance matrix S can be calculated by using the inverse Hooke's law:

$$S = \begin{bmatrix} \varepsilon_{xx}^{(A)} & 0 & 0 \\ 0 & \varepsilon_{yy}^{(B)} & 0 \\ 0 & 0 & \gamma_{xy}^{(C)} \end{bmatrix} \cdot \begin{bmatrix} \sigma_{xx}^{(A)} & \sigma_{xx}^{(B)} & \sigma_{xx}^{(C)} \\ \sigma_{yy}^{(A)} & \sigma_{yy}^{(B)} & \sigma_{yy}^{(C)} \\ \tau_{xy}^{(A)} & \tau_{xy}^{(B)} & \tau_{xy}^{(C)} \end{bmatrix}^{-1} \quad (10)$$

The superscripts A, B, C define the load schemes as presented in Fig.2 which were employed in order to calculate the stresses and strains. The compliance matrix of the orthotropic material in the plane stress condition is related to the parameters of the linear elasticity as follows:

$$S = \begin{bmatrix} \frac{1}{E_x} & -\frac{\nu_{yx}}{E_y} & 0 \\ -\frac{\nu_{xy}}{E_x} & \frac{1}{E_y} & 0 \\ 0 & 0 & \frac{1}{G_{xy}} \end{bmatrix} \quad (11)$$

Due to the symmetry of the compliance matrix 4 parameters E_x , E_y , ν_{yx} , G_{xy} are enough to define the elastic behaviour of the orthotropic material.

Explicit FE analysis is employed to obtain the failure limit of the material. The prescribed displacements as in Fig.2 applied on the edges of RS are increased with

constant velocity till the failure of the fibres or of the matrix material takes place. The prescribed displacement \mathbf{U}_t at time moment t is:

$$\mathbf{U}_t = t \cdot \Delta\mathbf{U}, \mathbf{U}_0 = 0 \quad (12)$$

$\Delta\mathbf{U}$ is a displacement increment per time unit.

The stresses $\sigma^{[t]}$ of the representative structure (RS) at the time moment t are calculated using the equation (9). To define the nonlinear stress and strain relation for the *MAT_LAMINATED_COMPOSITE_FABRIC (*MAT_058) material model in the finite element software LS-DYNA, longitudinal and transverse tensile strengths XT , YT and shear strength SC with the respective failure strains ε_{XT} , ε_{YT} and γ_{SC} are required.

The longitudinal tensile strength XT is the maximum stress $\sigma_{xx}^{[t]}$ of the RS loaded by the scheme A (Fig.2) before the failure (13). Similarly, the transverse tensile strength YT is the maximum stress $\sigma_{yy}^{[t]}$ (14) of the RS loaded by the scheme B (Fig.2) and the shear strength SC is the maximum shear stress $\tau_{xy}^{[t]}$ (15) of the RS loaded by the scheme C (Fig.2) if the RS is loaded till the failure occurs.

$$XT = \max_i(\sigma_{xx}^{[i]}); \varepsilon_{XT} = \varepsilon_{xx}^{[k]}, k: \sigma_{xx}^{[k]} = XT \quad (13)$$

$$YT = \max_i(\sigma_{yy}^{[i]}); \varepsilon_{YT} = \varepsilon_{yy}^{[k]}, k: \sigma_{yy}^{[k]} = YT \quad (14)$$

$$SC = \max_i(\tau_{xy}^{[i]}); \gamma_{SC} = \gamma_{xy}^{[k]}, k: \tau_{xy}^{[k]} = SC \quad (15)$$

$\sigma_{xx}^{[i]}$, $\sigma_{yy}^{[i]}$, $\tau_{xy}^{[i]}$ – the mean stresses of the RS at the i -th time moment, $\varepsilon_{xx}^{[i]}$, $\varepsilon_{yy}^{[i]}$, $\gamma_{xy}^{[i]}$ – the strains of the RS at the i -th time moment.

3. Numerical examples

3.1. Component materials

The matrix and fibre materials are assumed to be isotropic in the simulation of the mechanical behaviour of the UD composite defined by the heterogeneous FOM. The material model *MAT_PLASTIC_KINEMATIC (*MAT_003) is used in LS-DYNA for the matrix and fibre material simulation with parameters in Table 1. The fibres are oriented in the x direction.

Table 1. Parameters of the fibre and matrix materials.

	Fibre	Matrix
Mass density, kg/m ³	2.70E3	1.30E3
Young's modulus, N/m ²	7.50E10	3.10E9
Poisson's ratio	0.334	0.400
Yield stress, N/m ²	5.00E8	7.17E7
Tangent modulus, N/m ²	1.50E9	3.82E8
Failure strain	0.04	0.07
Shear modulus, N/m ²	2.81E10	1.11E9

3.2. Effective parameters

The FEM simulation scheme defined in Section 2.2 is employed for the square structure with different number of the fibres to evaluate the effective properties of the material. The structure is assembled of the FOM periodic elements (Fig.1, B). The structure is called representative (RS) if more fibres in the structure do not affect the effective parameters. The Young's modulus in the fibre E_x and transverse E_y directions and Poisson's ratio ν_{yx} are not affected by the number of the fibres in the analysed structure unlike the shear modulus G_{xy} if sides of the structure remain straight in the deformed shear configuration (see Table 2). This requirement has a significant effect for the stresses of the elements near the boundaries. As the shear modulus is evaluated by using the mean stress, the impact of boundary constraints is reduced by increasing the number of the fibres in the structure (see Table 2). If the number of the fibres increases, the waviness of the fibres occurs inside the analysed structure and the mean stress converges to the mean stress of the structure without constraints for the fibres. In order to accelerate convergence the mean stress of the periodic element at its centre can be used as the mean stress of the analysed structure. Similar results were obtained for the model if there is no requirement that sides of the element remain straight in the deformed configuration (Fig.3).

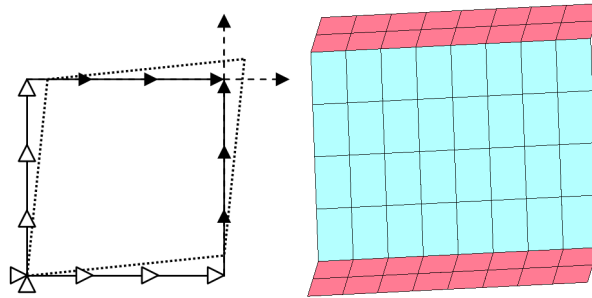


Fig.3. Shear load scheme and periodic element in the deformed configuration.

Table 2. Effective parameters of the structure assembled of different number of periodic elements.

Number of the periodic elements in the RS	$G_{xy}(A)$	G_{xy} (centre)	$G_{xy}(B)$	E_x	E_y	ν_{yx}
1 (1x1)	1.33E10	1.33E10	3.98E9	5.70E10	1.27E10	0.078
9 (3x3)	6.64E9	5.17E9	3.98E9	5.70E10	1.27E10	0.078
25 (5x5)	5.41E9	4.17E9	3.98E9	5.70E10	1.27E10	0.078
81 (9x9)	4.70E9	3.96E9	3.98E9	5.70E10	1.27E10	0.078
225 (15x15)	4.39E9	3.97E9	3.98E9	5.70E10	1.27E10	0.078
441 (21x21)	4.23E9	3.98E9	3.98E9	5.70E10	1.27E10	0.078

$G_{xy}(A)$ – the mean stress of the RS under shear load (Fig.2, C), $G_{xy}(\text{centre})$ – the mean stress of the periodic element in the centre of the RS under shear load (Fig.2, C), $G_{xy}(B)$ – the mean stress of the RS under shear load without straight side requirement (Fig.3) is used in evaluation.

Three sets of the homogeneous material parameters are considered in the numerical examples with the parameters in Table 3 for the material model *MAT_LAMINATED_COMPOSITE_FABRIC (*MAT_058) in LS-DYNA. Shear strength and corresponding strain are not evaluated analytically, but these values are required in the numerical model of the material. The fictitious shear strength and strain at shear strength values (Table 3, \diamond) are used for the stability of the homogeneous material model C. Two material models with parameters evaluated using the FEM approach are considered. Only the evaluation of the shear properties differ in both models. For the material model A, the shear modulus, shear strength and the strain at shear strength are evaluated using the pure shear assumption in simulation of the behaviour of the RS consisting 441 periodic elements. For the shear parameters of the material model B, there is no requirement that lines remain straight in the deformed shear configuration and shear modulus, shear strength and the respective strain are

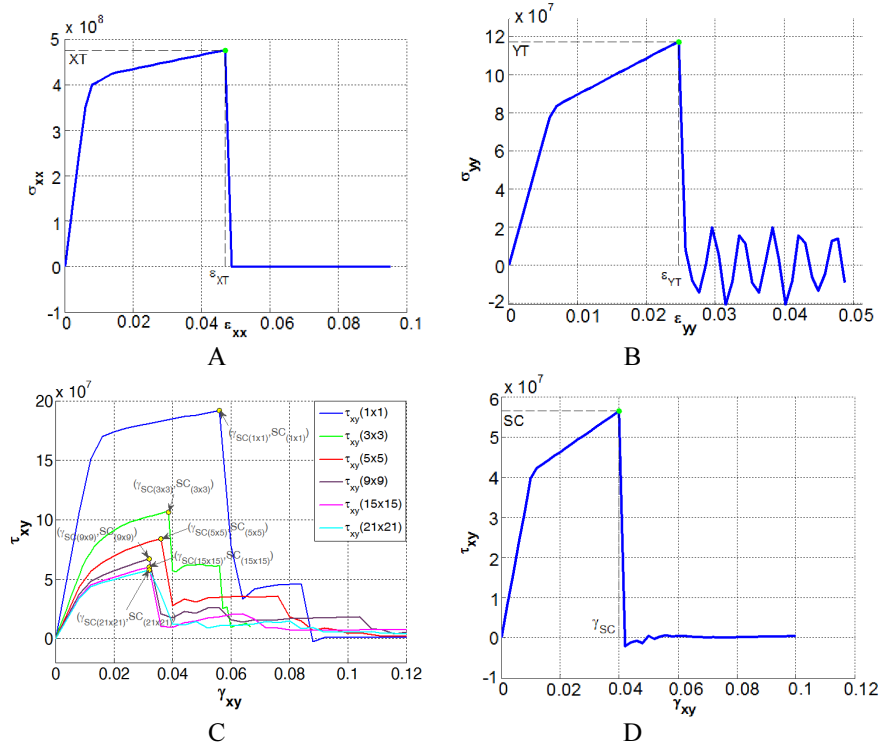


Fig. 4. True stress – true strain curves: (A) longitudinal tensile load; (B) – transverse tensile load; (C) – shear load (different size of the analysed structure, sides remain straight in the deformed configuration); (D) – shear load (no straight side requirement).

evaluated for the RS consisting one periodic element. The rule of mixture is employed to evaluate the mass density ρ of the homogeneous material:

$$\rho = \rho_f v_f + \rho_m v_m \quad (16)$$

ρ_f, ρ_m – mass density of the fibre and matrix material respectively.

Shear modules G_{yz}, G_{zx} are not evaluated for the FOM of the UD composite, but non-zero values are required to avoid numerical instabilities in the FEM analysis. Nilakantan et al. (2011) assume all three shear modulus are equal and the value is two orders of magnitude lower than the longitudinal modulus in simulation of the woven flexible fibres. In this research the evaluated shear modulus G_{xy} is applied and the G_{yz}, G_{zx} are three orders of magnitude lower than the evaluated shear modulus.

The longitudinal and transverse tensile strength with respective strains are evaluated for the structure of one periodic element. For the longitudinal tensile load (Fig. 4, A), both fibre and matrix materials reach yield points. The analysed structure fails completely after the fibres fail as most elements are deleted in the FEM model. For the transverse tensile load (Fig. 4, B), only the matrix material exhibits plastic behaviour. Moreover, the material is not weakened in the longitudinal direction – only elements representing the matrix material are deleted in the FEM model.

The simulated shear stress – strain curves depend on the size of the analysed structure (Fig. 4, C) if the sides remain straight in the deformed configuration. The shear strength used in the homogeneous material model A is the shear strength of the largest analysed structure (21x21) under the pure shear load. For the material model B the shear strength and the corresponding strain values are evaluated from the shear stress – strain curve (Fig. 4, D) for one periodic element using the load scheme in Fig.3 (no requirement that the sides remain straight in the deformed configuration).

For the homogeneous material models if longitudinal or transverse strength is reached the stresses are reduced to the stress equal to 1% of the maximum stress. The shell element is deleted if its effective strain is 100%.

Table 3. Parameters of the homogeneous material models.

	FEM approach		Analytical approach
	(A)	(B)	(C)
Mass density, kg/m ³	2.35E3		2.35E3
Young's modulus, E_x , N/m ²	5.70E10		5.70E10
Young's modulus, E_y , N/m ²	1.27E10		1.27E10
Poisson's ratio, ν_{yx}	0.078		0.078
Shear modulus, G_{xy} , N/m ²	4.09E9	3.98E9	3.97E9
Strain at longitudinal strength, ε_{XT}	0.047		0.040
Strain at transverse strength, ε_{YT}		0.025	0.018
Strain at shear strength, γ_{SC}	0.032	0.040	1◊
Longitudinal strength, XT , N/m ²	4.76E8		4.32E8
Transverse strength, YT , N/m ²		1.17E8	8.90E7
Shear strength, SC , N/m ²	5.78E7	5.64E7	3.97E9◊

The linear elastic parameters of the analytical model and material model B differ insignificantly. The longitudinal and transverse tensile strength values estimated

numerically are higher than the values computed analytically. Moreover, shear strengths for the material models A and B differ less than 10%. Although the shear strength values are close for both models but the shear strength for the material model A is reached at lower strain. This affects differences in the shear stress – strain curves of the models.

3.3. Biaxial tensile test for the 10x10 structure

The biaxial tensile load (Fig. 5) with augmentative displacements of the same magnitude in both directions is performed for the structure of 10x10 periodic elements using explicit FEM analysis. The 1st order model of the UD composite is analysed as the basis model. The mean stresses are compared for the structures with material models in Table 3.

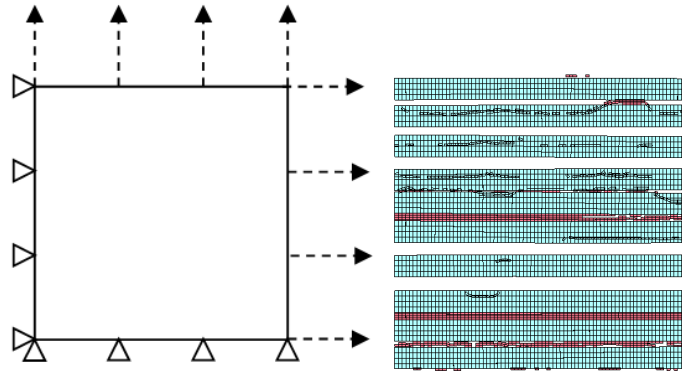


Fig. 5. Load scheme and the structure after the matrix failure.

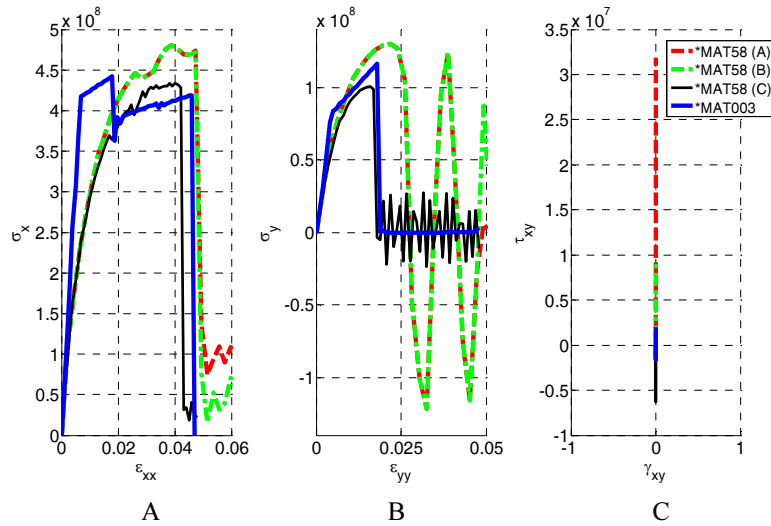


Fig. 6. Mean stresses of the structure (Fig. 5).

For the FOM (MAT003), the elastic to plastic transition of the matrix material causes the stress – strain relationship to change the slope and the stress in the transverse direction decreases to zero after the matrix material fails (Fig. 6, B). This causes the reduction of the mean stress in the longitudinal direction (Fig. 6, A). After the matrix failure, the fibres govern the behaviour of the composite until the fibre failure. As no shear strain occurs under this type of loading (Fig. 6, C), the behaviour of the homogeneous material models A and B differ insignificantly until the strength is reached in both directions. Significant shear stresses appear after the element failure in longitudinal and transverse directions and differ for the material models A and B due to the differences in shear parameters. Both models A and B overestimate the strength of the UD composite in the transverse and longitudinal directions. To the contrary, the homogeneous material model C underestimates the strength in the transverse and longitudinal directions.

3.4. Shear test for the 10x10 structure

The shear load (Fig. 7) with augmentative displacements is performed for the structure of 10x10 periodic elements with explicit FEM analysis. The FOM of the UD composite is analysed as the basis model. The mean stresses are compared for the structures with material models in Table 3.

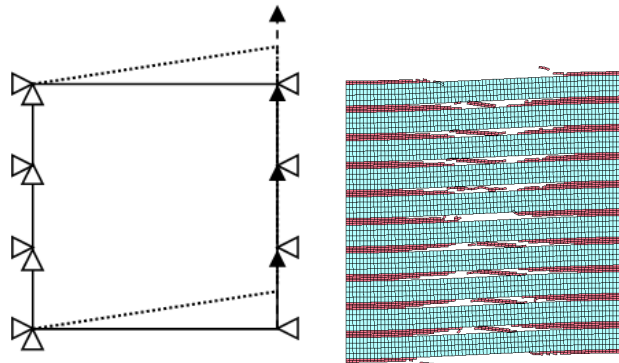


Fig. 7. Load scheme and the structure after failure.

Similarly to the previous example there are only two types of strain under this load (Fig. 8, B). The elastic to plastic transition of the matrix material causes the change of the slope in the shear stress – strain curve and shear stress reduction after the matrix failure (Fig. 8, C). The fictitious shear strength values were used for the material model C and the shear stress – strain relationship is linear until the failure. This model shows a good agreement with the basis model while the both matrix and fibre materials are elastic. The material model B underestimates the shear strength and strain. The material model A fails at higher strain than the basis model. As the shear behaviour is analysed, the stress – strain curve of the basis model depends on the number of the periodic elements and the constraints on the sides of the structure.

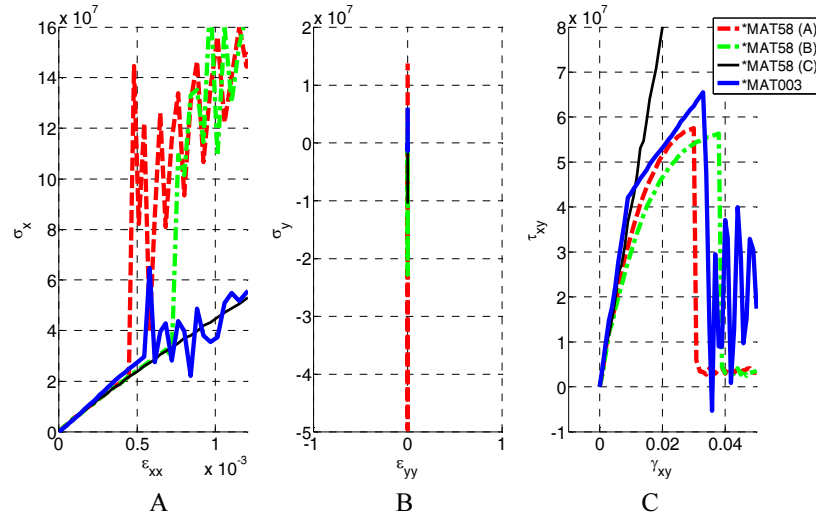


Fig. 8. Mean stresses of the structure (Fig. 7).

4. Final remarks

In this article, the effective elastic and failure parameters were evaluated analytically and using FEM simulations.

Using the homogeneous material model the 486 degrees of freedom were reduced to 24 degrees of freedom for the representative element. If the effective parameters are evaluated without the requirement that sides of the representative element remain straight, the linear elastic parameters differ insignificantly compared to the parameters evaluated analytically.

In the numerical examples the behaviour of the structures with different material parameters were compared. The parameters were evaluated analytically and numerically with the different boundary requirement for the shear test. It is enough to analyse longitudinal and transverse tensile tests for one periodic element to evaluate the effective Poisson's ratio and Young's modulus in the longitudinal and transverse directions. The shear properties of the FOM depend on the number of the fibres in the model if the sides of the model are straight after the deformation. If number of the fibres increases, shear properties converge to the shear properties evaluated for one periodic element without the straight – side requirement.

The behaviour of homogeneous material models with parameters evaluated numerically differ insignificantly if no shear behaviour is considered and overestimate strength of the UD composite in the longitudinal and transverse directions. As the fictitious shear values were used for the material model with parameters evaluated analytically, this material model describes linear shear behaviour only and underestimates strength in the longitudinal and transverse directions.

Micromechanical model with the ideal microstructure does not reflect micro-cracks, misaligned fibres and other defects. To consider these defects, the randomly distributed microstructure and higher order model should be analysed.

References

- Berger, H., Kari, S., Gabbert, U., Rodriguez-Ramos, R., Guinovart, R., Otero, J. A., Bravo-Castillero, J. (2005). An analytical and numerical approach for calculating effective material coefficients of piezoelectric fiber composites. *Int. J. Solids. Struct.* 42, 5692–5714.
- Kanit, T., Forest, S., Galliet, I., Mounoury, V., Jeulin, D. (2003). Determination of the size of the representative volume element for random composites: statistical and numerical approach. *Int. J. Solids. Struct.* 40, 3647–3679.
- Maimi, P., Camanho, P. P., Mayugo, J. A., Davila, C. G. (2007). A continuum damage model for composite laminates: Part I – Constitutive model. *Mech. Mater.* 39, 897–908.
- May, M., Nossek, M., Petrinic, N., Hiermaier, S., Thoma, K. (2014). Adaptive multi-scale modeling of high velocity impact on composite panels. *Compos. Part A-Appl. S.* 58, 56–64.
- Milton, G. W. (2002). *The Theory of Composites*. Cambridge University Press, Cambridge.
- Nilakantan, G., Keefe, M., Wetzel, E. D., Bogetti, T. A., Gillespie, Jr J. W. (2011). Computational Modeling of the Probabilistic Impact Response of Flexible Fabrics. *Compos. Struct.* 93, 3163–3174.
- Pelissou, C., Baccou, J., Monerie, Y., Perales, F. (2009). Determination of the size of the representative volume element for random quasi-brittle composites. *Int. J. Solids. Struct.* 46, 2842–2855.
- Pinho-da-Cruz, J., Oliveira, J. A.; Teixeira-Dias, F. (2009). Asymptotic homogenization in linear elasticity. Part I: Mathematical formulation and finite element modeling. *Comp. Mater. Sci.* 45, 1073–1080.
- Ribeiro, M. L., Tita, V., Vandepitte, D. (2012). A new damage model for composite laminates. *Compos. Struct.* 94, 635–642.
- Taniguchi, N., Nishiwaki, T., Kawada, H. (2012). Tensile strength of unidirectional CFRP laminate under high strain rate. *Adv. Compos. Mater.* 16(2), 167–180.
- Thibaux, P., Chastel, Y., Chaze, A. M. (2000). Finite Element Simulation of a Two Phase Viscoplastic Material: Calculation of the Mechanical Behavior. *Comp. Mater. Sci.* 18, 118–125.
- Vasiliev, V. V., Morozov, E. (2001). *Mechanics and Analysis of Composite Materials*. Elsevier Science, Oxford.
- Xing, Y. F., Yang, Y., Wang, X. M. (2010). A multiscale eigenelement method and its application to periodical composite structures. *Compos. Struct.* 92, 2265–2275.

Received January 8, 2015, revised February 19, 2015, accepted February 19, 2015